DOMAIN DECOMPOSITION BASED TWO-LEVEL NEWTON SCHEME FOR NON-LINEAR PROBLEMS

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There has been a vast amount of research done in the field of domain decomposition[1], most of which deals with linear problems. When solving non-linear problems the equations are linearized, and then the domain decomposition approaches are used to solve the ensuing linear problem. There are often applications however, where non-linear effects are restricted to small regions. The above mentioned schemes are not efficient in handling such localized non-linearities.

In this paper we develop a domain decomposition scheme for non-linear problems based on a two-level Newton approach. The scheme consists of a lower level Newton scheme to solve the sub-domain problems. As such, if a sub-domain problem behaves linearly, we need not assemble and factorize its stiffness matrix repeatedly. An upper level Newton scheme is used to solve the interface problem. This latter problem is obtained by enforcing the necessary kinematic and flux interfacial conditions. In our C^0 formulation we prescribe essential boundary conditions on the interface to satisfy the kinematic interfacial conditions. In this way we avoid the complications associated with floating sub-domains in lagrange multiplier based domain decomposition schemes. The interfacial flux condition is satisfied by balancing the reactive nodal fluxes, which result from the prescribed essential boundary conditions on the interface nodes. A similar approach has been used previously for coupling multi-physics problems[2, 5].

The developed approach has been implemented in parallel using MPI. The decomposition of the domain into several sub-domains is done by METIS© a graph partitioning routine. All sub-domain solves are performed in parallel, and we employ a sparse direct solver to factorize the sub-domain matrices. The sensitivities needed to assemble the interface problem involve a forward and back solve using the already factored sub-domain matrices. To solve the interface problem we use the iterative solvers present in PETSc©. The probing method is used to precondition the iterative solver. The formation and solution of the preconditioner are also done in parallel. Thus every stage of the proposed approach has been parallelized and shows good scalability.

References

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